In oder Aus Modern Improvements Upon and Applications of the Bloom Filter

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Abstract

Bloom filters are data structures used to determine set membership of elements, with applications from string matching to networking and security problems. These structures are favored because of their reduced memory consumption and fast wallclock and asymptotic time bounds.

Generally, Bloom filters maintain constant membership query time, making them very fast in their niche. However, they are limited in their lack of a removal operation, as well as by their probabilistic nature. In this paper, we discuss various iterations of and alternatives to the generic Bloom filter that have been researched and implemented to overcome their inherent limitations.

Bloom filters, especially when used in conjunction with other data structures, are still powerful and efficient data structures; we further discuss their use in industy and research to optimize resource utilization.

Introduction to Bloom filters

A Bloom filter is a probabilistic data structure used to test set membership queries in constant time. Queries may return false positives, but never a false negative (thus classifying them as a false-biased Monte Carlo algorithm). Standard Bloom filters include insert and set-membership query operations, and lack element removal, iteration, and other features common in binary search trees, hash tables, or more common data structures that can be used similarly.

Construction of a standard Bloom filter

A Bloom filter represents a set of n items, and consists of h unique hash functions and an array of m bits. To add an element to the Bloom filter, compute its hash with each of the h functions and set the bit at each index. (If the bit has been set previously, keep it set.) To query an element, compute its hashes and return "true" if all h bits are set; return false otherwise. Figure 1 demonstrates a query operation.

An element f not in the set could exist such that all h of its corresponding bits have been set by other elements. Querying f would return "true": a *false*

positive. A query will never mistakenly report that an element is not a member of the set when it actually is—in other words, there are no *false negatives*.



Figure 1: An example Bloom filter storing the set $\{x, y, z\}$ with h = 3 hash functions. x, y, and z are each mapped by three unique hash functions to three bits. w is queried by checking the three bits that it hashes to. Since one of w's corresponding bits is unset, the query will return "false."

False positives

The false positive rate of Bloom filters can be estimated [1] using

$$FPR = \left(1 - \left(1 - \frac{1}{m}\right)^{hn}\right)^h \approx \left(1 - e^{-\frac{hn}{m}}\right)^h,\tag{1}$$

where m is the size of the Bloom filter, h is the number of hash functions, and n is the number of elements inserted. The approximate form is graphed in Figure 2.

To fix the false positive rate at FPR, the optimal number of hash functions to use [2] is

$$h = \log_2\left(\frac{1}{\text{FPR}}\right). \tag{2}$$

Example use case

Bloom filters are often used to reduce memory or storage requirements of set membership data structures. An example problem in which Bloom filters could be used is browser vendors protecting users from malicious websites: while browser vendors could ship a list of known malicious URLs, shipping the full list would require excessively large network transaction and storage space. Instead, the browser vendor could include in the browser package a Bloom filter containing that list, and browsers could query the local Bloom filter quickly before loading any webpage. If the query returns "true", then the webpage can be checked



Figure 2: A plot of Equation 1: false positive rate vs. $\frac{n}{m}$, or the number of elements per bit of the Bloom filter, when h = 5. Note that Bloom filter false positive rates are ideally at most .02, where the rate of change is approximately 0—i.e., the first "few" (relative to m) elements inserted negligibly affect the false positive rate.

via the Internet against the exact list.¹ If the query returns "false", then the webpage is not on the list—a query for an element actually on the list will never return "false".

Theoretic advantages

The primary advantage of Bloom filters is how they markedly beat other data structures used for set membership queries in terms of memory overhead. Naive and deterministic implementations of set membership data structures (including binary search trees and hash tables) generally have to store the entirety of the elements that they represent. A significant improvement, Bloom filters represent an element with one bit per h hash functions. (Typically, fewer than 10 bits are required to represent an element in a Bloom filter [5].) Furthermore, the number of elements in the filter does not need to correlate with the number of bits (or the size of the array) used to store that element.

There are some extensions to the common Bloom filter that make this advantage even more extreme (notably the Fuzzy-Folded Bloom filter, which can support \sim 1.9 times the elements of a standard Bloom filter while maintaining the same false positive rate and a constant time complexity in all operations).

A secondary advantage of Bloom filters is their ability to insert and query elements in constant time. Binary search trees can do neither in constant time. Hash tables can often only query elements in amortized constant time, and generally have a linear worst case for insertion. The latencies of Bloom filter operations are solely dependent on the number and complexities of the hash functions it employs.

Because of Bloom filters' ability to quickly process elements with low space cost, they are particularly useful in problems that involve data streams [6] (or massive sets of elements with no defined upper bounds on size). For example, a massive hash table that is used to test set membership will eventually fill up and break when a boundless number of elements are inserted. While the load factor of a Bloom filter in the same scenario may become dangerously high, the core functionality of the data structure will always remain intact, even when inserting an extremely large number of elements.

Limitations

The probabilistic nature of Bloom filters may make them unsuitable for certain tasks—though that can be mitigated by choosing an appropriate size of the bit array and number of hash functions to achieve sufficiently low false positive rates. By Equation 1, one can increase the size m and manipulate (depending on $\frac{n}{m}$) the number of hash functions h to decrease the false positive rate to an acceptable percentage. To eliminate false positives, the Bloom filter can be used as a preliminary check to eliminate negatives before checking against the exact list. (This is faster than checking everything against the exact list, since Bloom

¹In fact, this is how Google Chrome protects users from malicious sites. As of 2010, the full list contained ~ 1 million websites, stored in an only 18 Mb Bloom filter [3] [4].

filter queries are in constant time and Bloom filters are small enough to fit in faster, limited-quantity memory (e.g., cache).)

The inability to remove elements from a Bloom filter makes the structure unfit for highly dynamic and volatile sets, where the membership of elements changes rapidly. (Even re-inserting elements into a new Bloom filter doesn't work, since a Bloom filter cannot efficiently and precisely report the entire set; the hashing of elements and subsequent setting of bits is an irreversible operation.) However, there are several extensions on the common Bloom filter which add removal support—these will be discussed in the next section.

Developments

In this section, we will discuss a few iterative improvements on the standard Bloom filter, as well as the dynamic Fuzzy-folded Bloom filter and the "practically better" Cuckoo filter. These improvements include addition of a removal operation, superior space complexity, and better hardware interaction in the interest of practicality. While all of these solve the same general sorts of problems, one may be better than another for specific use cases. Table 1 compares several properties of the discussed filters.

Counting Bloom filter

The most primitive Bloom filter iteration supporting element removal is the Counting Bloom filter. In this variation, each bucket or bit is replaced by a number of bits (usually four), which serve as a counter. If an inserted element hashes to an index, that index is incremented. If an element is removed, then all the counters it hashes to are decremented. When querying an element, the query returns "true" if all the counters it hashes to are greater than 0; the query returns false otherwise. Disadvantages of this variation relative to the standard Bloom filter include the possibility of bit overflow (in the counters) and higher memory overhead (as it requires storing several bits for each array index instead of only one).

Blocked Bloom filter

Blocked Bloom filters (like standard Bloom filters) do not support removal. However, they are constructed to exhibit high spatial locality by fitting several smaller Bloom filters individually into cache lines. They guarantee less than two cache misses on negative queries [7], where standard Bloom filters can have up to h misses on negative queries. Thus, a Blocked Bloom filter can be a much more practical data structure than a general Bloom filter. However, for these improvements, implementing a Blocked Bloom filter requires some knowledge of the hardware.

d-left Counting Bloom filter

d-left Counting Bloom filters use fingerprinting to support removal. They insert elements by computing their *d*-left hash to store as a fingerprint—removal is accomplished simply by deleting the computed fingerprint. In this way, they are almost more similar to a standard hash table than a bloom filter. This structure's space cost is up to twice as expensive as that of a standard Bloom filter, but it requires half the space of a Counting Bloom filter [5]. The idea of fingerprinting incorporated in this structure is also a concept fundamental to Quotient and Cuckoo filters.

Quotient filter

Quotient filters are another extension of the standard Bloom filter which also use string fingerprinting to support removal. The main difference between Quotient filters and *d*-left Counting Bloom filters is how Quotient filters use an extension on standard hashing called "quotienting" to efficiently hash and reconstruct the fingerprints of elements. In quotienting, the high and low bits of a computed fingerprint are partitioned; the low bits are then stored in a bucket indexed by the high bits. Due to this scheme, collisions generally display high spatial locality and therefore are hardware-friendly [8]. However, this structure still relies on linear probing in order to resolve collisions. Thus, Quotient filters cannot give the same guarantees on asymptotic time complexity as most other Bloom filter implementations, and its performance suffers at $\geq 75\%$ load [2].

Table 1: Characteristics of Bloom filters, extensions, and variations. The cache misses column shows the worst-case. In the General Bloom and the Blocked Bloom designs, h indicates the number of hash functions used. In the *d*-Left Counting Bloom construction, *d* is the number of partitions in its hash table [2]. As explained in the following section, *f* is the number of completed fuzzy-folds.

| Filter type | Space cost | Cache misses per lookup | Deletion support |
|-------------------------------|----------------------------|-------------------------|------------------|
| General Bloom | $1 \times$ | h | No |
| Counting Bloom | $3 \times \sim 4 \times$ | h | Yes |
| Blocked Bloom | $1 \times$ | 1 | No |
| <i>d</i> -Left Counting Bloom | $1.5 \times \sim 2 \times$ | d | Yes |
| Quotient | $1 \times \sim 1.2 \times$ | ≥ 1 | Yes |
| Fuzzy-Folded Bloom | $\sim 0.5 \times$ | $\overline{h}(f+2)$ | No |
| Cuckoo | $\leq 1 \times$ | 2 | Yes |

Fuzzy-folded Bloom filter

A Fuzzy-folded Bloom filter describes the continuous compression ("folding") of two standard Bloom filters (created from a bipartition of the original array) each of size $\frac{m}{2}$ bits into a single, compressed filter with $\frac{m}{2}$ buckets. Exactly half of the space of the original array is allocated to this compressed Bloom filter, and the other half is used to support two new Bloom filters each of size $\frac{m}{4}$ bits [6].

In this, the Fuzzy-folded Bloom filter is a "dynamic" Bloom filter, growing to maintain a low false positive rate while accommodating further insertions. The fuzzy-folding operation does not break the invariants of the original filter (in that it will never introduce the possibility of false negatives), nor does it increase the false positive rate of the filter [6].

The fuzzy-folding operation overlays bits at the same position in both Bloom filters and uses fuzzy logic in each bucket of the product array to represent the compression of these filters. This process (and therefore compressed bit representation) is non-commutative; it is necessary to logically reconstruct the ordering to effectively query the filter [6].

Insertion is similar to standard Bloom filter insertion. There are always two non-compressed Bloom filters in the array. There is also a designated load threshold, applicable to both non-compressed filters. If the first array has yet to reach this threshold, h bits are set in the first array (whose indices are determined by the element's h hashes). If the first array has reached this threshold, h bits are set in the second array. If both arrays have reached this threshold, then they are fuzzy-folded, and two new Bloom filters are allocated with $m/(2^{2+f})$ bits each (where f denotes the number of fuzzy-fold operations conducted before this operation, starting from 0).

Queries operate by first checking the second non-compressed filter (and returning "true" when a standard Bloom filter would). If there is no match, the first non-compressed filter is then checked. If there is again no match, then the filters are sequentially queried from most recently to least recently compressed. Therefore, in the worst case, the time complexity of queries is linear in terms of the number of fuzzy-folds performed. Also, in all cases, the speed of queries suffers drastically compared to that of a standard Bloom filter when the size of the compressed arrays becomes sufficiently small [6].

The most notable advantage of Fuzzy-folded Bloom filters is how they can accommodate roughly 1.9 times the elements of a general, space-optimized Bloom filter while maintaining the same false positive rate [6]. In practice, this is a desirable trade-off, considering linear query complexity in terms of number of fuzzy-folds is not a significant limitation, or even comparable to linear in terms of inserted elements.

Iterative Patterns

Bloom filter evolutions are built with practicality in mind: due to the realworld performance boost from cache optimization, many of these filters are constructed for high spatial locality. Likewise, many datasets are volatile, and so fingerprinting is commonly used to let a Bloom filter variant support removal without introducing unacceptable space overhead.

Cuckoo Filter

Cuckoo filters use many of the same paradigms and ideas of Bloom filters, fill the same niche (of performing highly space and time efficient set membership queries with no false negatives), yet approach a few key concepts in ways that differentiate them from Bloom filters.



Figure 3: An illustration of the Cuckoo hashing technique used by Cuckoo filters. Each element has two possible buckets. Inserting into an occupied bucket displaces the occupying element into its alternate bucket. E.g., inserting an element that first hashes to where C is would cause the new element to displace C to C's alternate bucket. Inserting an element that first hashes to where A is would displace A into the bucket B currently occupies, displacing B into its empty alternate bucket. Inserting an element that first hashes to the bucket W would cause an infinite loop since W and H hash to the same two buckets.³

Notably, Cuckoo filters support element removal. They accomplish this by using a fingerprinting scheme similar to that seen in *d*-left Counting Bloom filters and Quotient filters, where deleting an element amounts to deleting its fingerprint. Specifically, removing an element from a Cuckoo filter is accomplished by checking both of the element's possible buckets and deleting the fingerprint if it exists in either [2].

A Cuckoo filter is also more capable of taking advantage of its allocated space. Because of Cuckoo filters' insertion scheme (illustrated in Figure 3), which sets a fingerprint in only one (compared to h) bucket per insertion and (in some implementations) allows buckets to contain multiple fingerprints, a load factor of 95% without a marked increase in false positives is very achievable [2].

The final primary advantage of Cuckoo filters is in their simple design and construction. Compared to more complicated variations on Bloom filters that reduce time and space complexity at comparable margins, such as Blocked Bloom

 $^{^3 \}rm Original figure by Ramus Pagh, via Wikipedia, under a Creative Commons License (CC BY-SA 3.0). Figure has been rotated.$

filters and Golomb-Compressed Sequences [2], the concepts and implementation details behind Cuckoo filters are relatively simple.

On the other hand, Cuckoo filter insertion is arguably worse than that of a standard Bloom filter. The process of insertion has the same worst case as general Cuckoo hashing, where all buckets for the hashed fingerprint are occupied, leading to a chain reaction of displacements throughout the entire table. Nevertheless, Cuckoo filters still maintain amortized $\mathcal{O}(1)$ insertion [2].

Cuckoo filters also have an upper bound on the number of times one fingerprint can be inserted: if the Cuckoo filter has buckets of size b, then elements with the same fingerprint can be inserted at most 2b times [2]. Disabling the removal operation can overcome this limitation, but removal is a sought-after feature in this niche.

To maintain an acceptable false-positive rate, fingerprint size must scale with the size of the filter (or its number of buckets). Nevertheless, this is generally acceptable, as Cuckoo filters are more space efficient than the standard Bloom filter at low false positive rates ($\sim 3\%$) [2].

Bloom Filters in Practice

We give an in-depth discussion of a new *de novo* genome assembler, ABySS 2.0, which utilizes Bloom filters to trivialize memory requirements without sacrificing speed or accuracy, and increases DNA sequencing throughput. This and further advancements could revolutionize preventative healthcare, and affect the lives of many. We continue to list many of the ways Bloom filters are used in everyday life to make efficient otherwise difficult procedures.

De novo genome assembly

Genomics research—the field of characterizing genomes to better understand similarities and differences among species, or even individuals—has seen much development in the past decade in part thanks to growing and planned personalized medicine initiatives.⁴ During this time, the DNA sequence throughput of the industry's best instruments has constantly increased [1]. In particular, sequence assembly, which refers to aligning and merging fragments read or copied from a longer DNA sequence in order to reconstruct the original sequence,⁵ has seen

⁴"Personalized medicine" is a medical model which tailors medical decisions, practices, interventions and/or product usage to the individual patient's predicted risks. Because of this focus on the individual, and given the large part genetics play in individual health, the field depends on sequencing genomes *en masse*. The spread of personalized medicine may be a major boon to preventative healthcare, especially as sequencing becomes faster and more affordable.

 $^{^{5}}$ DNA sequencing technology, biological or synthetic, cannot read whole genomes (on the order of 7 billion basepairs split among 23 pairs of chromosomes in humans) in one pass. Instead, genome sequencing works by copying or listing the bases in a short (20–30000 bases) "reads" and then combining them. *De novo* sequence assembly, which constructs genomes without a backbone or template, combines these reads using overlaps as indicators of originally adjacent sequences. Figure 4 illustrates the *de novo* assembly process.

drastic reductions in both time and spatial requirements with the incorporation of Bloom filters and related algorithms in assemblers.



Figure 4: Sample sequence showing how a sequence assembler would take short fragments (pink, red, blue, and green) and match by overlaps to recreate the black sequence. Notice that the pink fragment could be in one of two positions in the sequence.⁷

The genome assembler ABySS is example of the efficiency of Bloom filters in de novo genome assembly: whereas ABySS 1.9 (which didn't use a Bloom filter) could assemble the human genome in 14 h using a whopping 418 GB of memory (across many machines), ABySS 2.0 can, using the same parameters, assemble the human genome in 20 h with a mere 34 GB⁸ [1]. ABySS 2.0 achieves this performance increase by consolidating its usage to a single machine (eliminating the need for messaging) and instead represents a De Bruijn graph using a Bloom filter. In this context, a De Bruijn graph stores all 4^k possible length-k sequences (called "k-mers", which are length k portions of the longer read) made up of symbols (bases) from {A, C, G, T} in the vertices

$$V = \{ (A, ..., A, A), (A, ..., A, C), ..., (A, ..., A, T), (A, ..., C, A), ..., (T, ..., T, T) \}$$

and all four possible "successor" sequences (where the first base is removed, the rest shifted left, and another base is appended) are represented in the edges

 $E = \{((v_1, v_2, \dots, v_n), (v_2, \dots, v_n, s_i) : i = 1, \dots, m)\}.$

 $^{^7\}mathrm{Figure}$ (presented without modification) by "Luongdl", via Wikipedia, under a Creative Commons License (CC BY-SA 3.0).

 $^{^{8}}$ The absolute time for sequencing doesn't matter: this is about throughput. This is an order of magnitude increase in throughput given the same resources, which matters for real-world applications. Roughly 10 times the number of patients can have their DNA sequenced using this sort of algorithm.

 $^{^{10}{\}rm Figure}$ (presented without modification) from [1], under a Creative Commons License (Attribution 4.0 International license)



Figure 5: An overview of the ABySS 2.0 assembly algorithm. A: A read is split into k-mers and loaded into the Bloom filter by computing the hash of each k-mer sequence and setting the corresponding bit in the Bloom filter. B: A path through the De Bruijn graph is traversed by querying all possible successor k-mers and advancing to those found. C: ABySS 2.0 trims dead-end branches and continues only along those at least a fixed length. The term "solid read" denotes a confirmed sequence. Edges are discovered in both directions: both predecssors and succesors are searched for [1].¹⁰

As described in Figure 5, ABySS 2.0 represents the vertices of the De Bruijn graph with a Bloom filter by setting bits corresponding to the vertices (k-mers). These k-mers are the length-k sequences of a short read, so in this way the Bloom filter contains the entire read. With all reads stored in the Bloom filter, ABySS 2.0 combines them to reconstruct the original sequence. The algorithm repeatedly queries the Bloom filter to discover De Bruijn graph edges (which lead to predecessor or successor k-mers). Since k-mers are relatively short and are only part of the read, this process may lead to representation of false edges in the graph; fortunately, using look-ahead mechanisms, these branches are trimmed if they do not continue for more than k nodes. (This look-ahead mechanism increases graph traversal cost, but eliminates the need for additional data structures.) As reads extend and more likely represent a correct path in the De Bruijn graph, they are branded "solid reads" and considered to be part of the original sequence. Through this process, the entire genome may be reconstructed from short reads with high accuracy.¹¹

Table 2 shows how ABySS 2.0 compares to other genome assembly algorithms in resource consumption. Notably, the tools which efficiently represent the De Bruijn graph (ABySS 2.0, MEGAHIT, Minia, and SGA) require much less memory than those that do not. BCALM¹², while extremely resource light, sacrifices sequence contiguity compared to the rest of of the listed assemblers. ABySS 2.0, using Bloom filters, achieves a marked improvement over ABySS 1.9 and the rest of the pack in efficiency, while achieving contiguity results on par with DISCOVAR denovo and ABySS 1.9. Note that Minia also uses Bloom filters, and ABySS 2.0 is largely based on Minia, with three novel features: (i) the use of solid reads, (ii) a look-ahead mechanism to eliminate false positives (as opposed to a seperate data structure), and (iii) a new hashing algorithm designed for DNA/RNA sequences. The authors of ABySS believe that there is still great opportunity for improving throughput without sacrificing contiguity—i.e., the algorithmic ideas of BCALM could be adapted to produce a more contiguous result [1]. (And perhaps Bloom filters are not the absolute most efficient means of representing a De Bruijn graph.)

| Assembler | Memory (GB) | Time (h) |
|----------------|-------------|----------|
| ABySS 1.9 | 418 | 14 |
| ABySS 2.0 | 34 | 20 |
| DISCOVARdenovo | 618 | 26 |
| BCALM | 5 | 9 |

Table 2: Peak memory usage and wallclock runtime with 64 threads of assemblies of GIAB HG004. Data from [1].

¹¹The accuracy is "on-par with other assemblers" [1], and need not be perfect; even the biological process of DNA transcription introduces error. Additionally, DNA has some protections against error in that many codons encode the same meaning, as well as the fact that much of the genome has little-to-no known effect.

 $^{^{12}{\}rm BCALM}$ uses a novel method of partitioning the De Bruijn graph to be resource efficient, and does not use Bloom filters.

| Assembler | Memory (GB) | Time (h) |
|------------|-------------|----------|
| MEGAHIT | 197 | 26 |
| Minia | 137 | 19 |
| SGA | 82 | 65 |
| SOAPdenovo | 659 | 35 |

Networking

Bloom filters are used in device discovery: if two previously paired devices meet again under different circumstances, they can skip pairing again (which would be unnecessary, since they have previously done so). Devices build a Bloom filter of devices they have paired with, and when attempt to connect with another device, send the list to the second device. If the second device recognizes one of its identifiers in the Bloom filter, it responds to the first device that they have paired previously, which initiates the mutual connection. This process, described and patented in [9], is used in Qualcomm devices (i.e., many or most cell phones and other portable devices) for ad-hoc network discovery (e.g., Bluetooth, WiFi direct, 802.xx wireless LAN).

Making the world go 'round

Facebook [10] uses Bloom filters to represent the social graph for typeahead search in order to display friends and friends-of-friends of the user's query. The Bloom filter uses 16 bits per friend connection (or graph edge).

Yahoo! Mail [11] uses Bloom filters to represent email contact list since the Bloom filter can fit in browser cache. This obviates the need for round-trip connections to Yahoo for verifying delivered emails are from contacts.

Tinder [12] [11] uses Bloom filters to record "right swipes" (accepting a user as a possible match) in order to remove users from the incoming list. When eventually the list is refreshed for new users to swipe on, those previously right swiped on will be filtered out. Some unseen users are filtered out by this process, but, as the saying goes, there are plenty of fish in the sea.

URL shorteners [11] employ Bloom filters to generate unique URLs: if a shortened URL has been previously used, it exists in the Bloom filter. Thus, by querying the Bloom filter with different shortened URLs until receiving a "false" response, the service can ensure unique URLs.

YouTube uses Bloom filters [11] to ensure recommended videos are not in the user's watch history, in addition to the algorithms that optimize for relevance metrics, to feed users new and interesting content.

Conclusion

The general Bloom filter is a powerful data structure thanks to its simplicity and its time and memory efficiency. Guaranteed constant time insert and query operations are rare among data structures, and often worth the trade-off of false positives in set membership problems. A more serious limitation for particular problems is the lack of a removal operation, which is overcome by various evolutions of the Bloom filter, and the Cuckoo filter. For many problems, one of these evolutions or alternatives may be better suited depending on the circumstantial resources and constraints.

Bloom filters, though simple and often hidden behind-the-scenes, have a profound and increasing effect on the modern world. This data structure makes efficient dating, entertainment, and networking—both social and digital—possible, as well as leading healthcare to be able to personalize treatment to the individual.

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